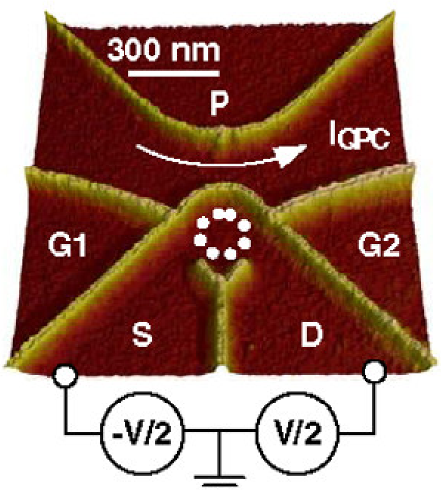
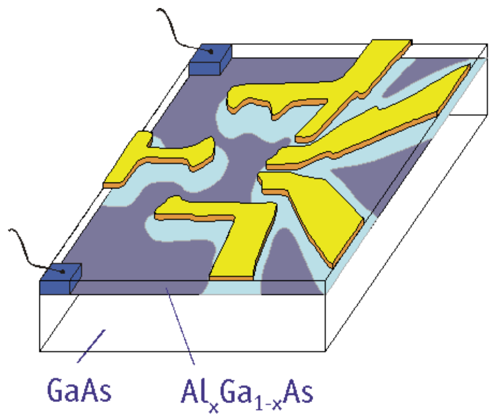
Electron nano-transport: why is it interesting?

Mikhail Polianski
NBI, Denmark

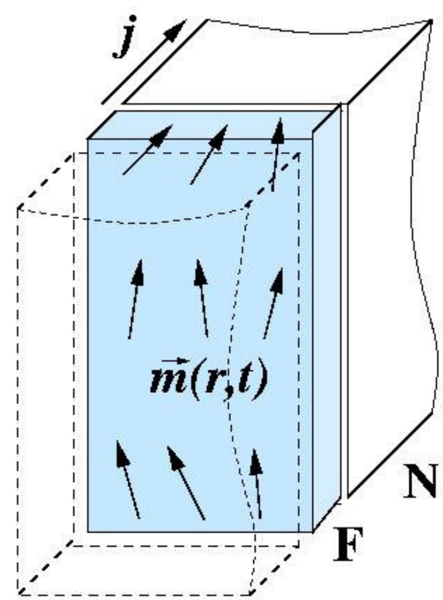
charge



interference



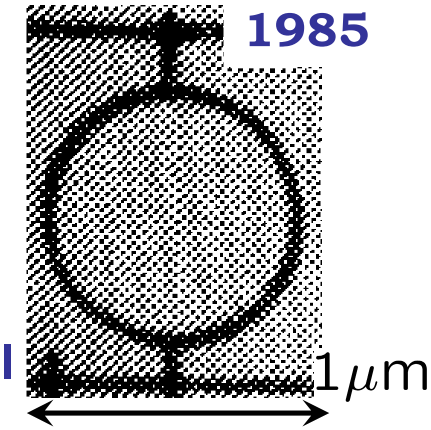
spin



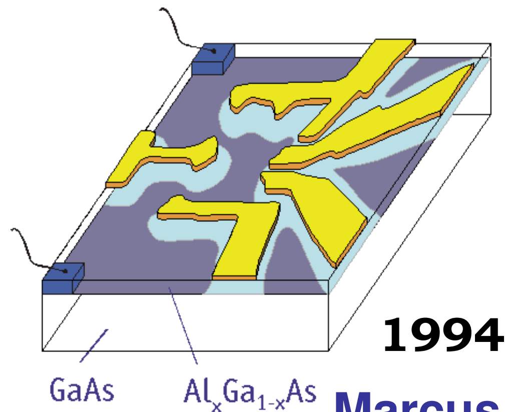
11 March, 2009

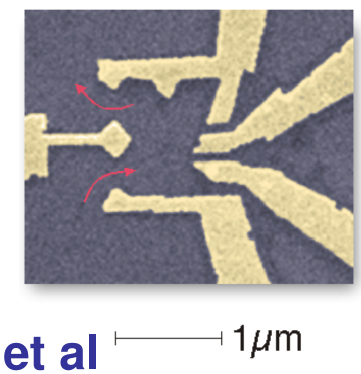
Outline

- Introduction. What is big/small? What is “mesoscopic”?
- Classical vs quantum
- 2 kinds of fluctuations
- AC transport
- Spintronics: who needs it?
- Conclusions

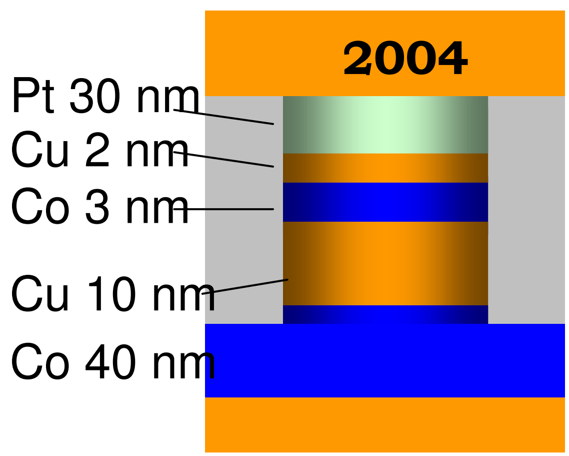


Webb et al

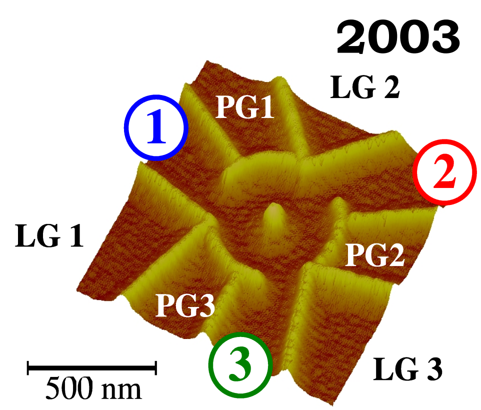




Marcus et al



Kiselev et al



Leturcq et al

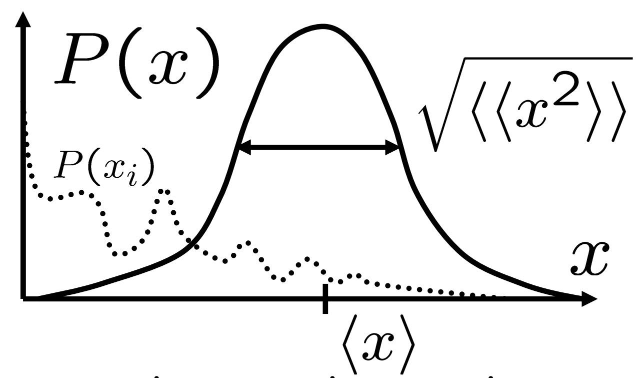
What do we need to know?

- Sum of many uncorrelated quantities with some unknown distributions is Gaussian (CLT)

$$P(x) \propto \exp\left(-\frac{(x-\langle x\rangle)^2}{2\langle\langle x^2\rangle\rangle}\right)$$

$$\langle x^2\rangle = \langle x\rangle^2 + \langle\langle x^2\rangle\rangle$$

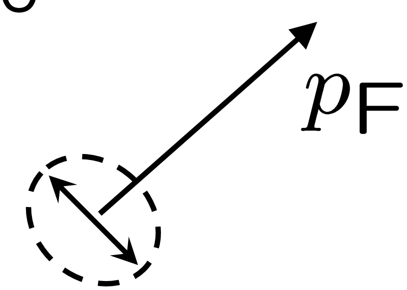
average width



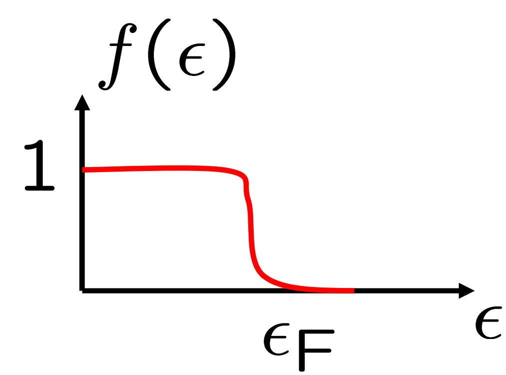
$$x = x_1 + x_2 + \ldots + x_N$$

- Fermi distribution (Pauli principle)
- Uncertainty relation

$$\Delta x \cdot \Delta p > \hbar \neq 0$$

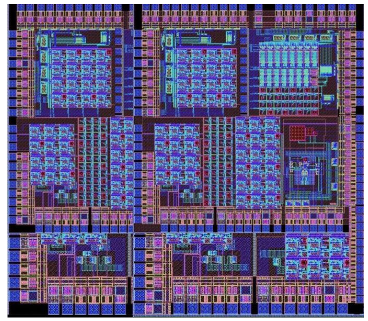


$$\Delta x \sim h/p_F = \lambda_F$$



Motivation

Faster operation
stable read/write
low heating
small size.



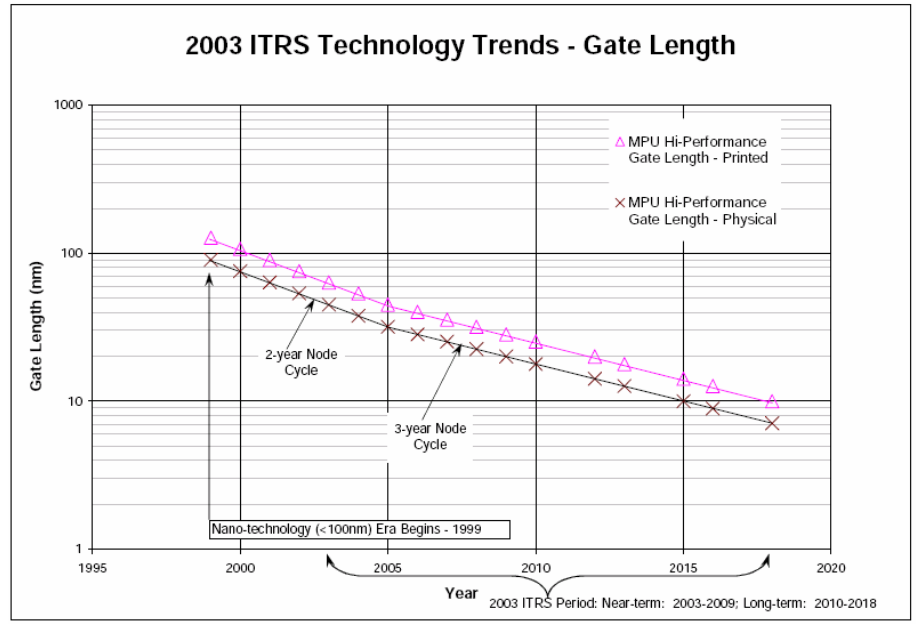
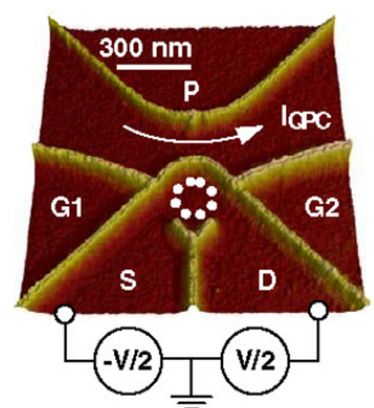


Figure 8 2003 ITRS—Gate Length Trends

Currently: $L \sim 10$ nm
Mean free path $l \sim 100 \mu$m

Reproducible (industry and science)

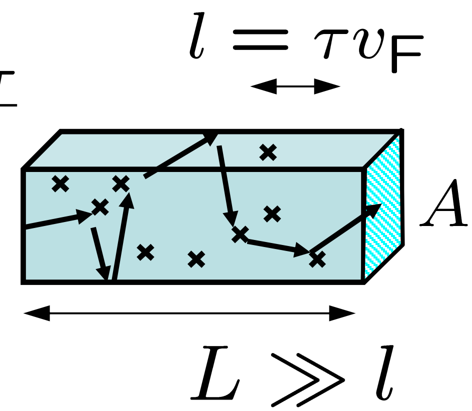
1. What IS small? 10 nm– is it small already?
2. Can we predict properties of small samples
Unpredictable: is it good or bad?



“Big” is classical physics scale

- Ohm’s law

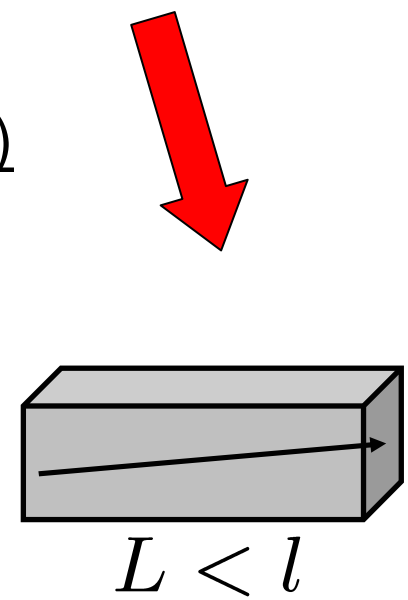
$$G = \frac{I}{V} = \sigma\frac{A}{L},\ \sigma \sim \frac{nev}{E} \sim ne\frac{Ee\tau/m}{E} \sim \frac{ne^2\tau}{m}$$



- Conductivity is material property, does not fluctuate
- $\hbar = 0$
  Electrons are point-like particles

This reproducibility fails when $L < l \sim 100\mathrm{nm}$?

In other words, diffusive (dirty) $\longrightarrow$ ballistic (clean)



Not conductivity σ, but sample’s conductance

$$\boxed{G = \frac{I}{V}}$$

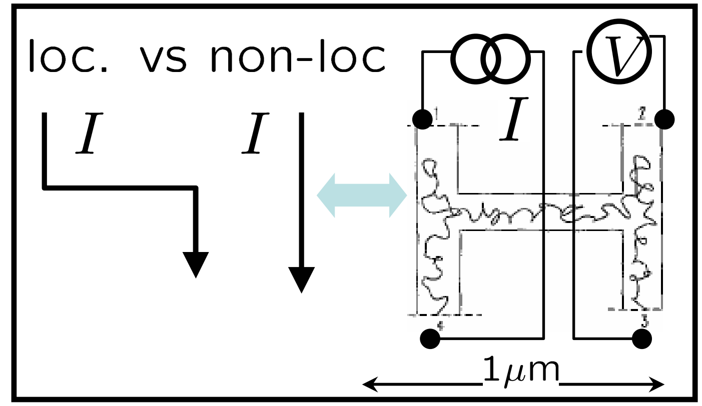
Not so simple: local vs non-local

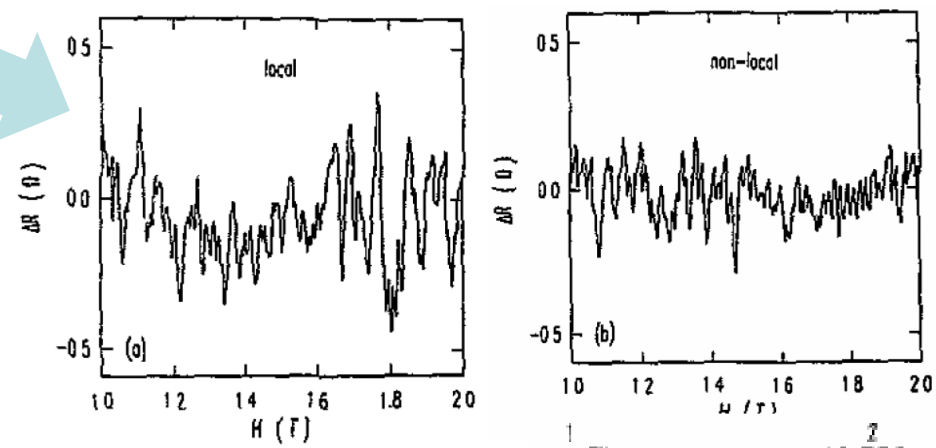
Classically, non-local response is suppressed, so are fluctuations $\propto \exp(-L/W), l \ll W \ll L$

But fluctuations are similar for local and non-local

Length l did not matter?

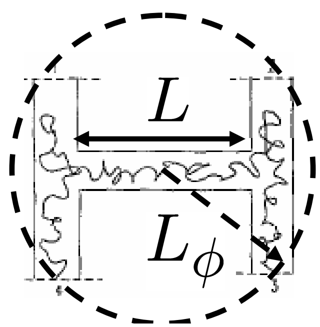
Haucke'90

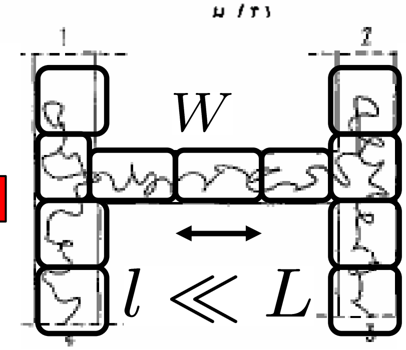




Introduce a new scale L_ϕ

When $l \ll L < L_\phi$, our classical intuition is wrong

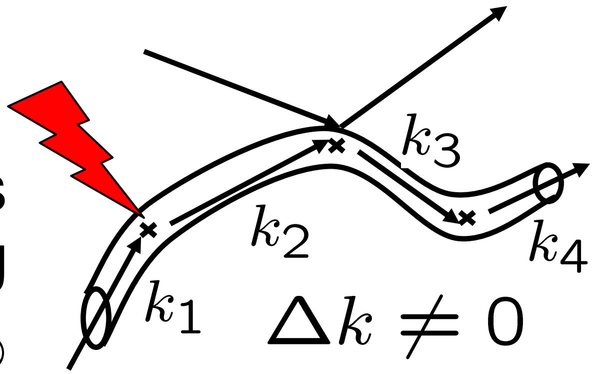




Quantum on larger scale

- Electrons gain phase, $\phi = \int \vec{k} d\vec{r}$
- On dephasing length L_ϕ electron loses phase memory due to inelastic scattering

$$L_\phi(T) \propto T^{-p}; T \rightarrow 0 \Rightarrow L_\phi \rightarrow \infty$$



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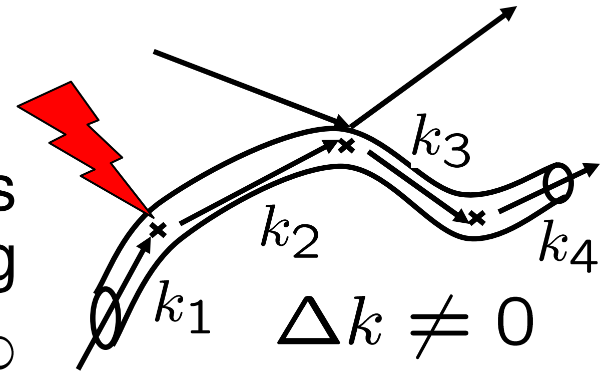
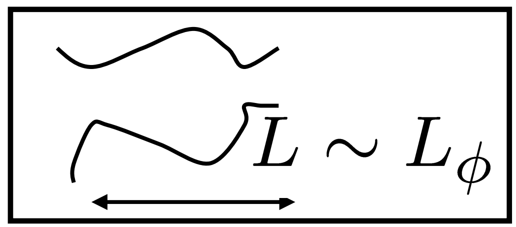


$\Delta k \neq 0$

Mesoscopic $L < L_\phi$ ⇨ strong fluctuations



"random error"

$g \pm \delta g$

$$r = \frac{1}{g}, \delta r \sim \frac{\delta g}{g^2}$$

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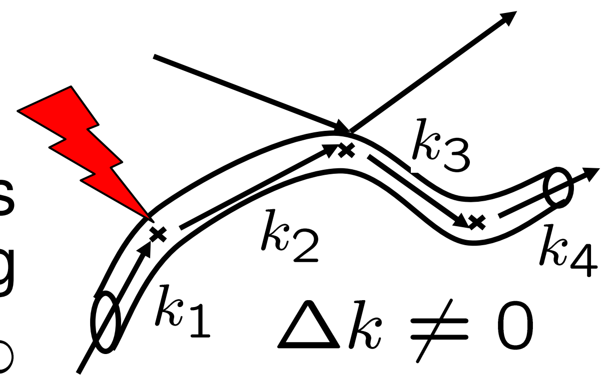


Mesoscopic $L < L_\phi$ ⇨ strong fluctuations

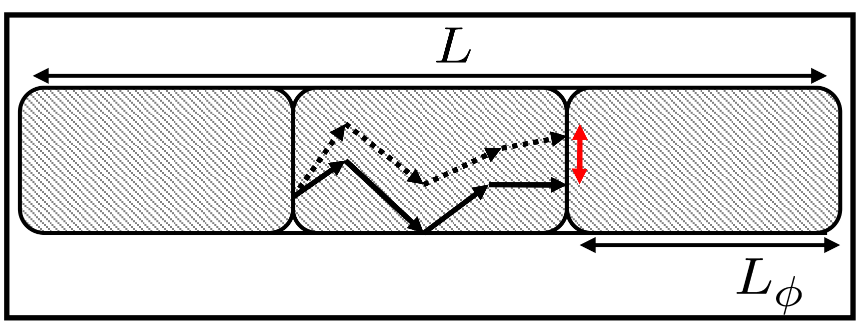
Macroscopic $L \gg L_\phi$ ⇨ sum of many uncorrelated resistors

$N \sim L/L_\phi \gg 1$

"random error"

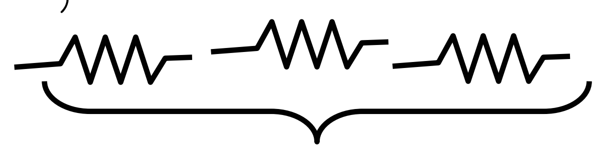
$g \pm \delta g$

$r = \frac{1}{g}, \delta r \sim \frac{\delta g}{g^2}$



⇔

$r, \delta r$



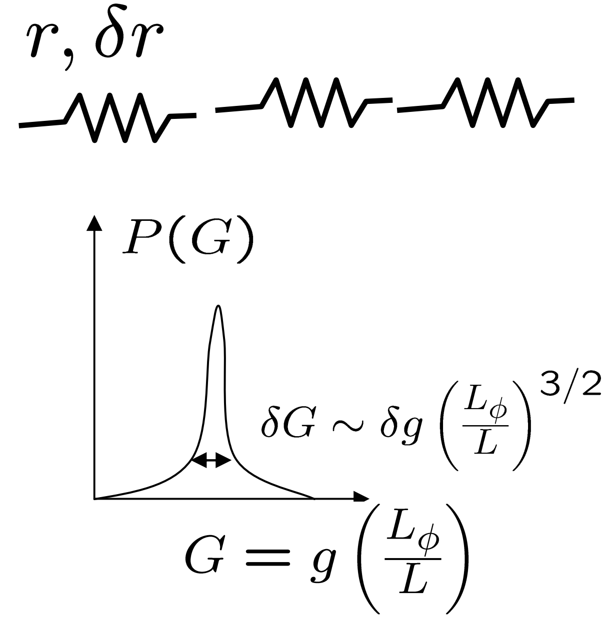
$R, \delta R = ?$

Mesoscopic scale

In classical limit, sum of "random errors" grows slower then average

$$R \propto rN, \delta R \sim \sqrt{\Sigma_i (\delta r_i)^2} \propto \delta r \sqrt{N}$$

$$\frac{\delta G}{G} \sim \frac{\delta g}{g}\sqrt{\frac{1}{N}} \ll \frac{\delta g}{g}$$



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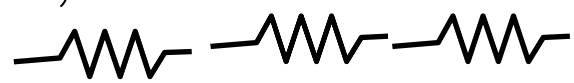

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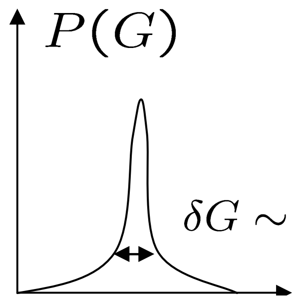
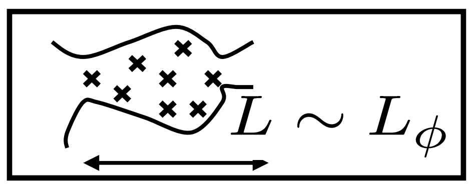


$\delta G \sim \delta g \left(\frac{L_\phi}{L}\right)^{3/2}$

$G = g\left(\frac{L_\phi}{L}\right)$

Mesoscopic fluctuations

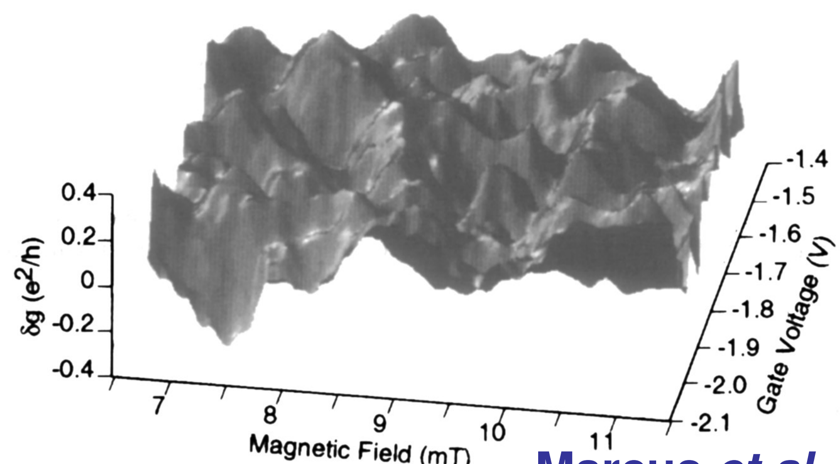
universal (UCF) $\delta g \sim e^2/h$



Fluctuations probe interference

$$T \sim 4K \Rightarrow L_\phi \sim 1\mu\text{m}$$

Take $T \longrightarrow 0$ and see quantum effects



Marcus *et al*

Not always point-like particles

$$G_{AB} \propto \frac{e^2}{h} \left| \sum_{i=1}^{N} A_i e^{i\phi_i} \right|^2$$

- Classical + interference contributions

$$G \propto \sum_i A_i^2 + 2 \sum_{i<j} A_i A_j \cos(\phi_i - \phi_j)$$

- Independent e-tubes $\sim \lambda_F < 50$ nm

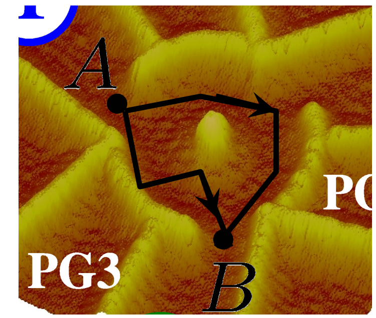
  $N \sim (W/\lambda_F)^{d-1}$, $d = 2, 3$

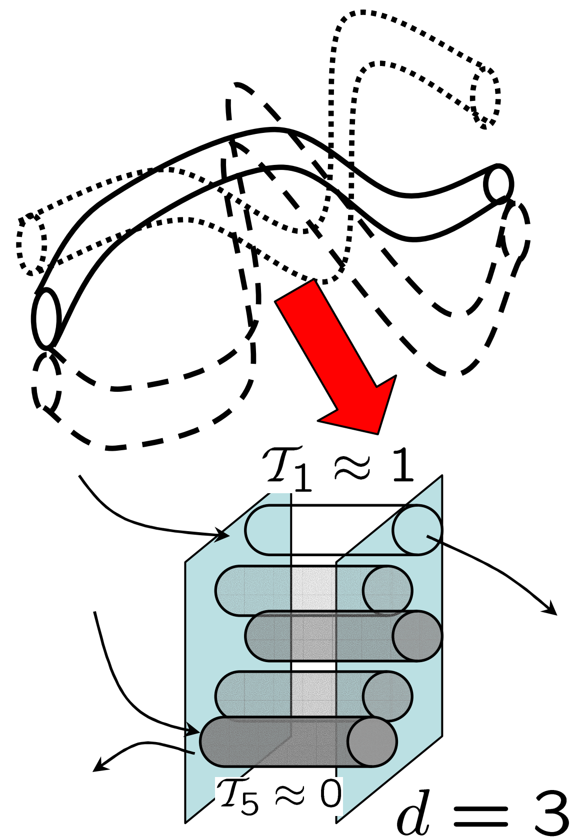
Tube=conduction channel with some transparency $\mathcal{T}_i$, $0 < \mathcal{T}_i < 1$

Channels similar, but conduct differently

Open channel $\mathcal{T} = 1$ perfect transmission

Closed channel $\mathcal{T} = 0$ insulator



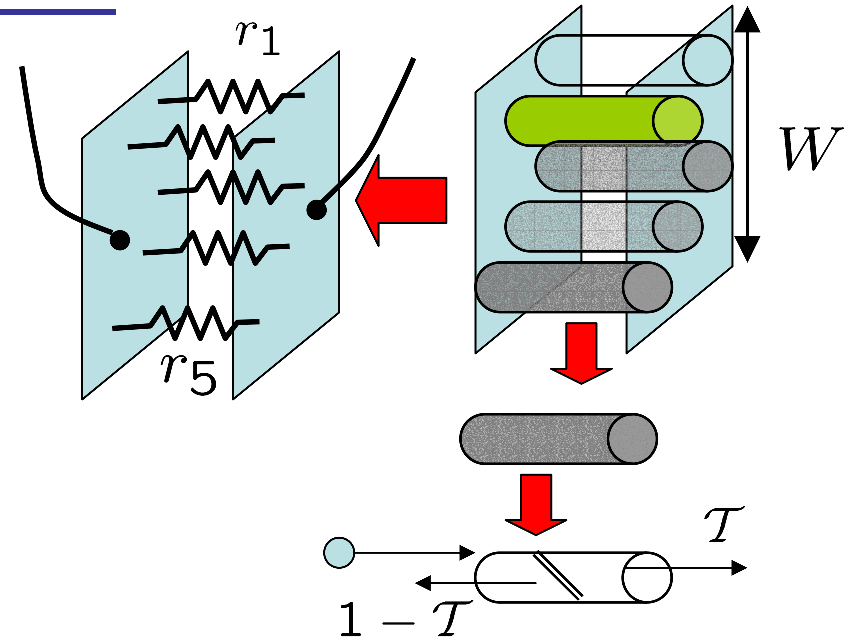


Channels

Find $G = \frac{1}{R} = \sum \frac{1}{r_i}$

Landauer formula

$$G = \frac{e^2}{h} \sum_{i=1}^{N} \mathcal{T}_i, \; N \sim \left(\frac{W}{\lambda_{\mathrm{F}}}\right)^{d-1}$$



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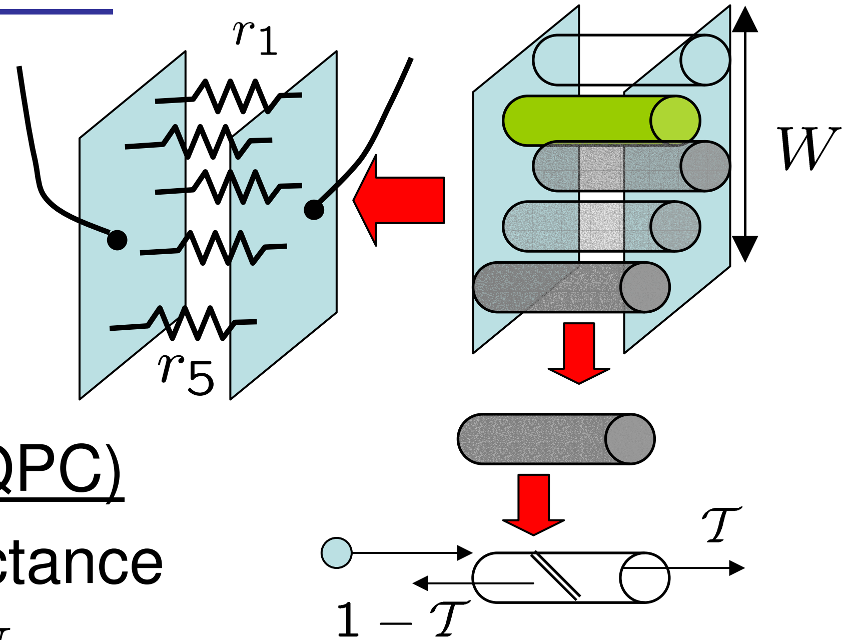


Ballistic Quantum Point Contact (QPC)

Classical contact (Sharvin) conductance

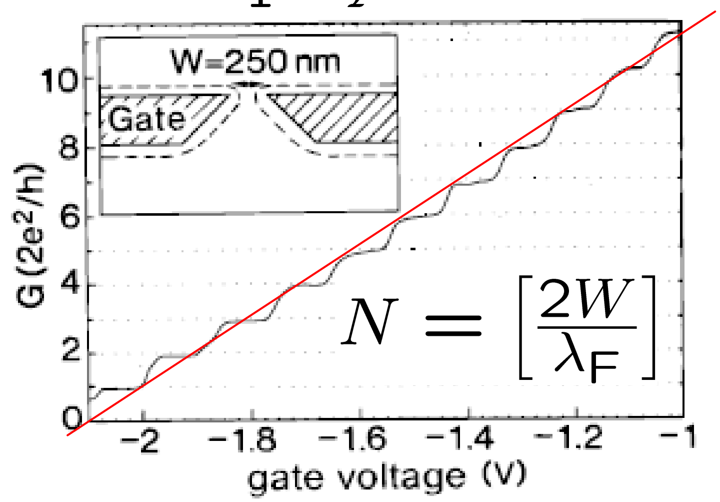
$$G_c = \frac{e^2}{h} \begin{cases} W k_F/\pi & d=2 \\ S k_F^2/4\pi & d=3 \end{cases} = \frac{N}{26k\Omega}$$

Conductance quantization (steps)

Ideal channels not always possible

Set $\{\mathcal{T}_i\}$ is sample's PIN

Easier with only one channel…



van Wees '88

Poisson in one channel

Uncorrelated events

Schottky'1918

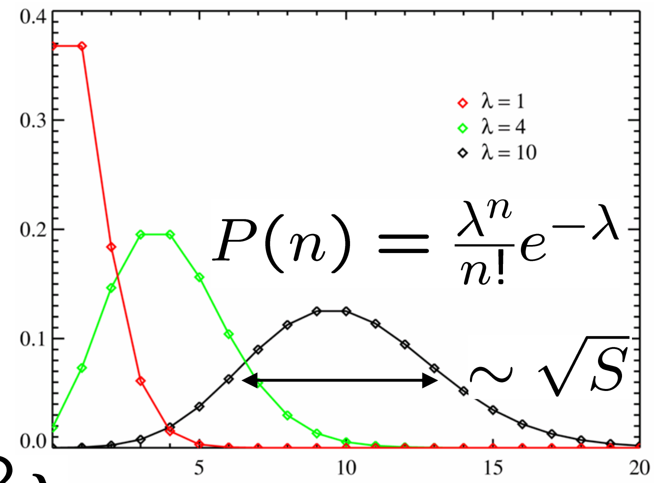
$$\langle q\rangle = e\sum_{n=0}^{\infty} nP(n) = e\lambda$$

$$\langle q^2\rangle = e^2\sum_n n^2P(n) = e^2(\lambda+\lambda^2)$$

$$\langle\langle q^2\rangle\rangle = \langle q^2\rangle - \langle q\rangle^2 = e^2\lambda$$

Average charge $q = e\lambda = \bar{I}t$

Shot noise–distribution width, $S_S = e^2\lambda$



$$P(n) = \frac{\lambda^n}{n!}e^{-\lambda}$$

$$\sim \sqrt{S}$$

$$\lambda = \bar{I}t/e$$

Fano factor, noise-to-signal ratio

$$\mathcal{F} = \frac{S}{e_0 I} = 1$$

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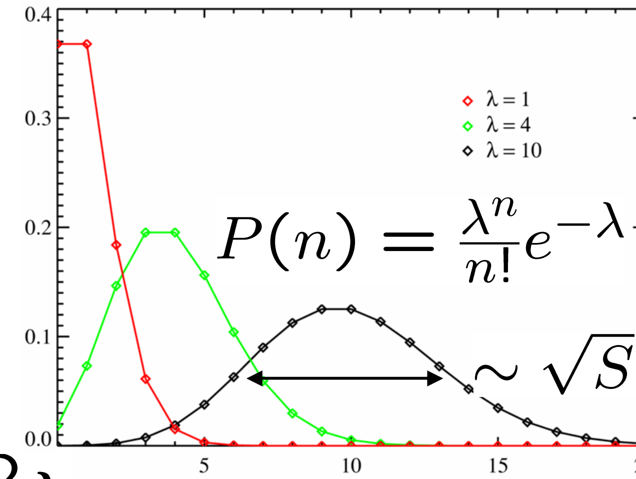

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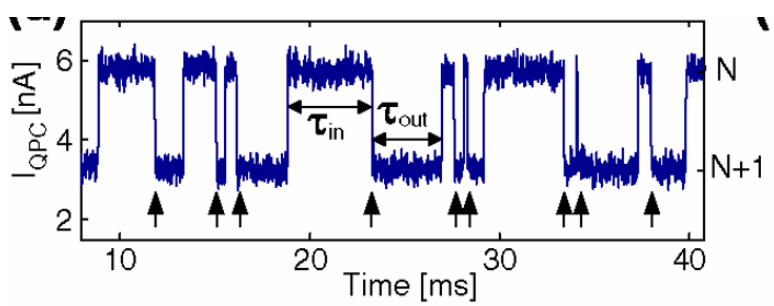
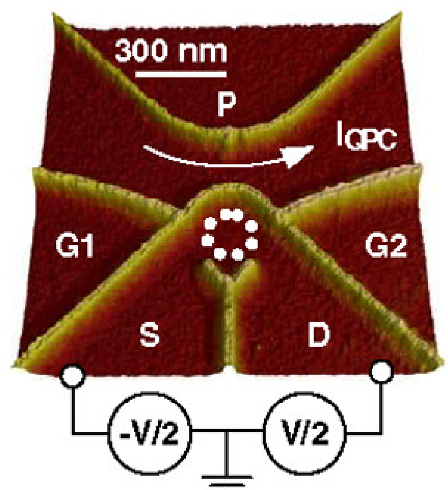
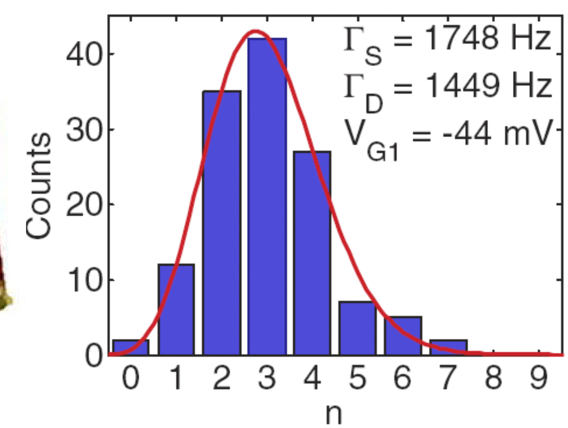






Gustavsson *et al* '06

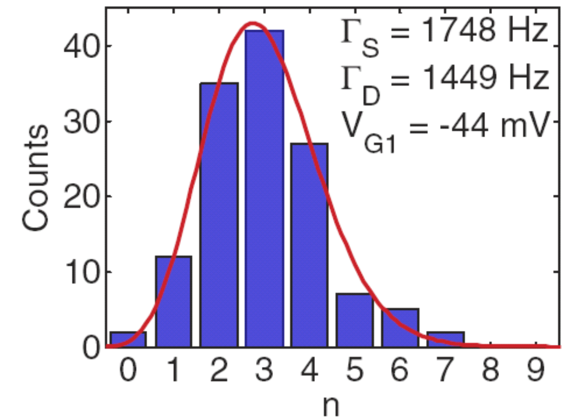
Good channels (metals)

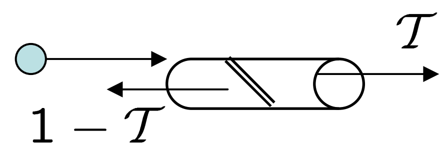
- **Unpredictable results**
- During time t make $N = eVt/h$ attempts with n successes

$$P(n|N) = C_N^n \mathcal{T}^n (1-\mathcal{T})^{N-n}$$

$$\langle q \rangle = e \sum_{n=0}^{N} n P(n|N) = eN\mathcal{T}$$

$$\langle q^2 \rangle = (eN\mathcal{T})^2 + e^2 N\mathcal{T}(1-\mathcal{T})$$





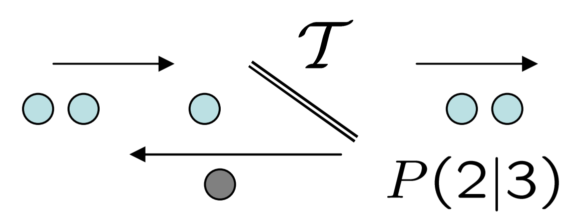
Shot noise is sub-Poissonian!

$$S_S = e^2 N\mathcal{T}(1-\mathcal{T})$$

$$\boxed{\mathcal{F} = 1 - \mathcal{T} < 1}$$

Good $\mathcal{T}$ – very different from classics

Not **rare** anymore!



$$P(2|3) = 3\mathcal{T}^2(1-\mathcal{T})$$

Various fluctuations

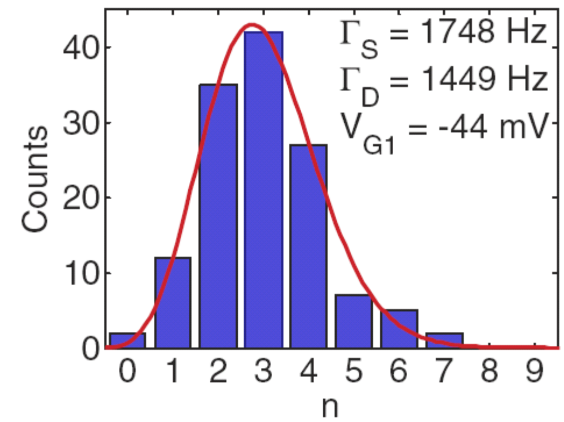
- Shot noise

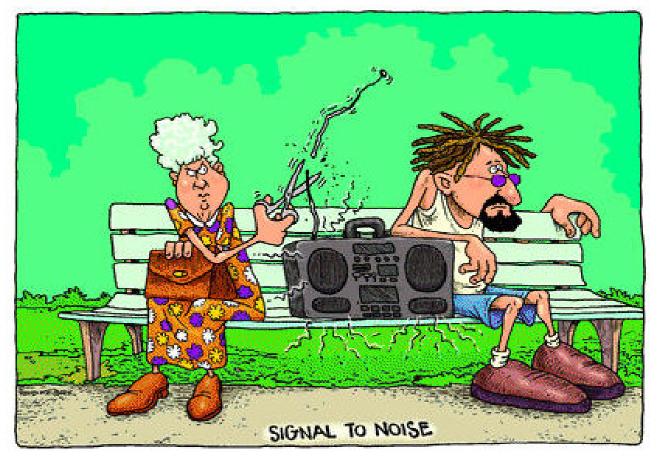
✓ Results unique for each experiment

✓ Noise is super-Poissonian for photons, sub-Poissonian for electrons

$$\mathcal{F}_{ph} = 1 + \mathcal{T} \qquad \mathcal{F}_e = \frac{q}{e_0}(1 - \mathcal{T})$$

✓ F.f. measures charge $q \neq e_0$





The noise is the signal

R. Landauer

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✓ F.f. measures charge $q \neq e_0$

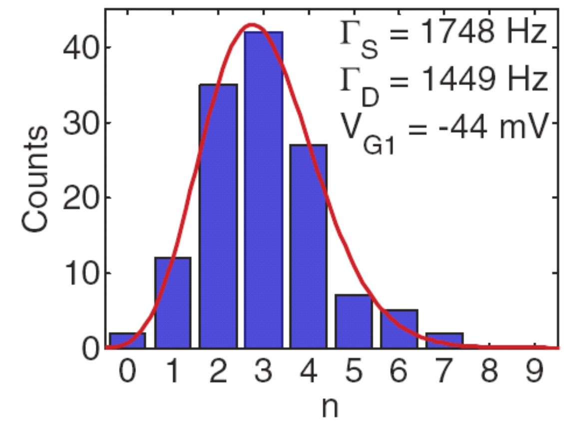
Now:

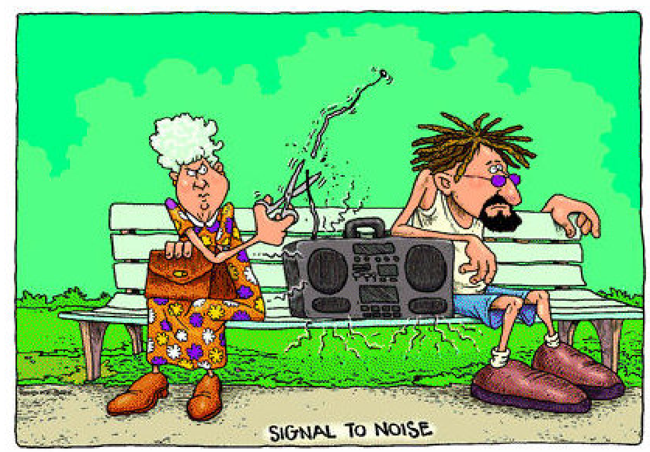
- Mesoscopic fluctuations

✓ Set $\{\mathcal{T}_i\}$ is sample's PIN

✓ Many channels give averaged $\mathcal{F}$

Different distributions of $\{\mathcal{T}_i\}$ give different results





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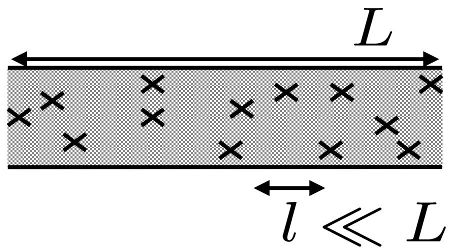
R. Landauer

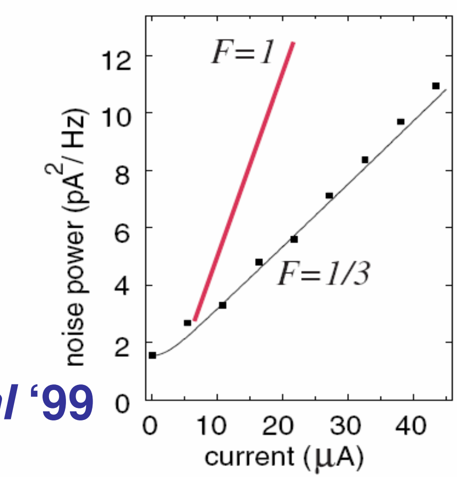
Fano factor in diffusive wires

$$\mathcal{F} = \left\langle \frac{\sum \mathcal{T}(1-\mathcal{T})}{\sum \mathcal{T}} \right\rangle$$



Conductance estimate: $\mathcal{T} \sim l/L \ll 1$
Poissonian (rare events) expected in diffusive wires, $\mathcal{F} = 1$

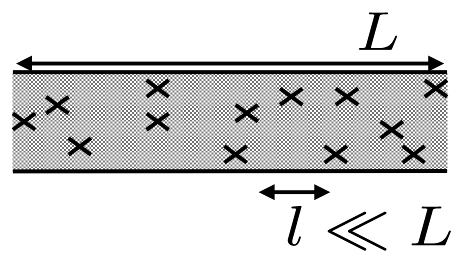
Fano factor was found very close to 1/3 (**SURPRISE?**)



Henny *et al* '99

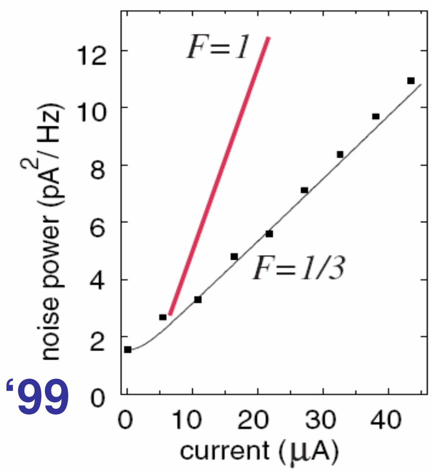
Fano factor in diffusive wires

$$\mathcal{F} = \left\langle \frac{\sum \mathcal{T}(1-\mathcal{T})}{\sum \mathcal{T}} \right\rangle$$



Conductance estimate: $\mathcal{T} \sim l/L \ll 1$
Poissonian (rare events) expected in diffusive wires, $\mathcal{F} = 1$

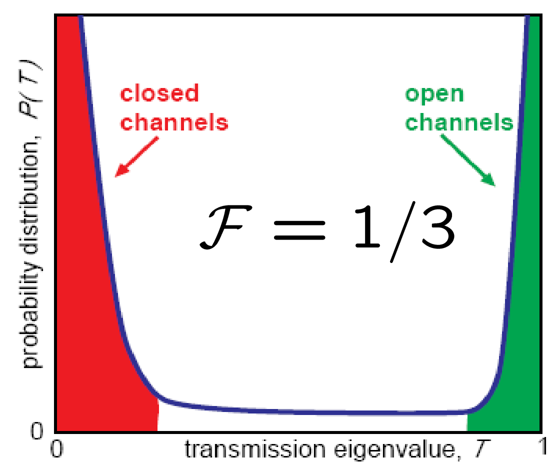
Fano factor was found <u>very close to 1/3</u> (**SURPRISE?**)



Henny *et al* '99

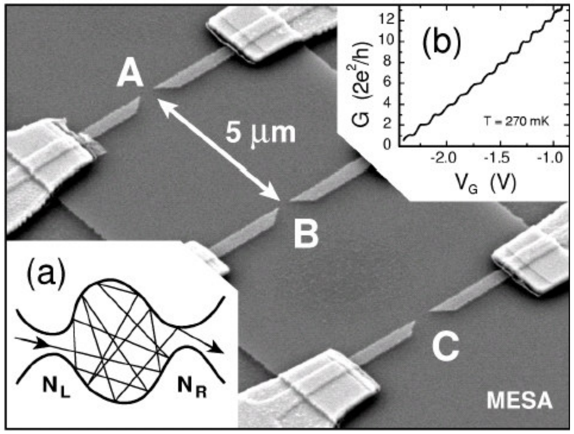
Contrary to naïve ideas: open channels with $\mathcal{T} \rightarrow 1$ do exist even if $l \ll L$

$$\rho(\mathcal{T}) = \frac{h\langle G \rangle}{2e^2} \frac{1}{\mathcal{T}\sqrt{1-\mathcal{T}}}$$

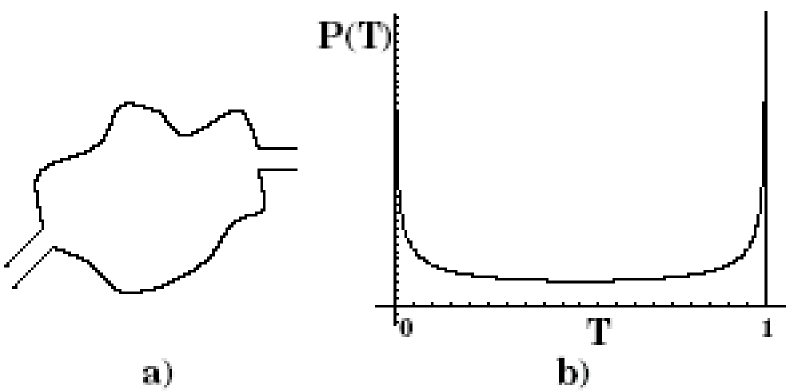


Beenakker-Büttiker'92, Nagaev'92

Quantum dots



$$\rho(\mathcal{T}) = \frac{N}{\pi\sqrt{\mathcal{T}(1-\mathcal{T})}} \Rightarrow \mathcal{F} = \frac{1}{4}$$



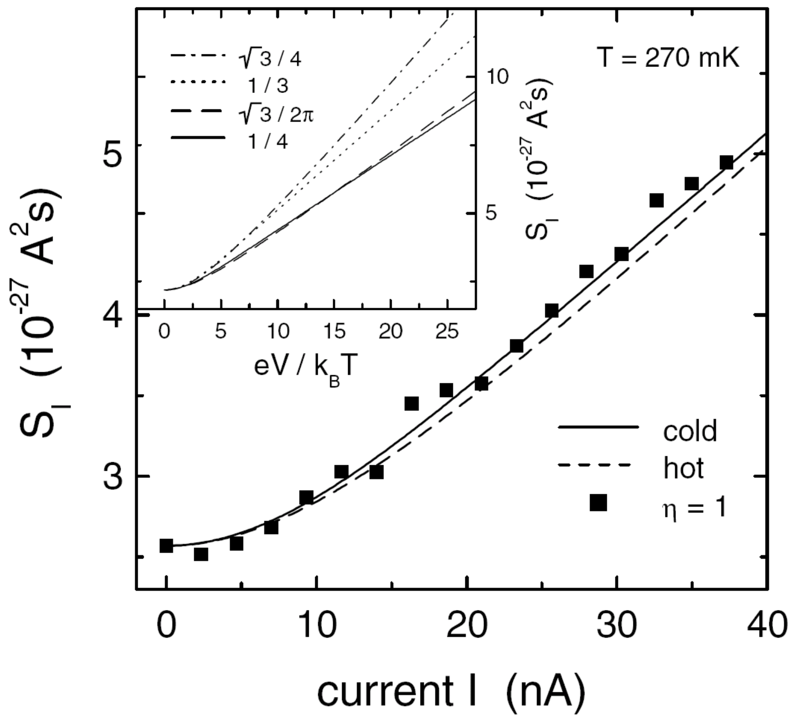


FIG. 3. Shot noise of a symmetric cavity and theoretical predictions for cold (solid line) and hot electrons (dashed line). Inset: comparison of the noise of a chaotic cavity ($1/4$ and $\sqrt{3}/2\pi$) with a diffusive wire ($1/3$ and $\sqrt{3}/4$) for cold and hot electrons.

Oberholzer *et al* '01

So, everything is clear?

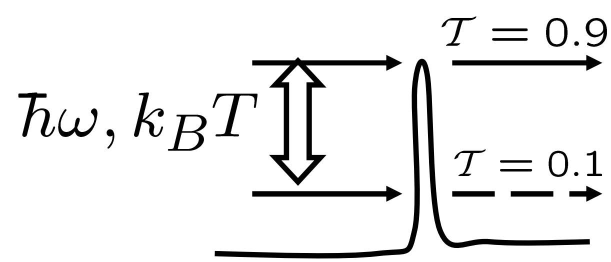
- We forgot too many things…

Electrons interact!
Nonzero temperatures
(useless Nyquist noise)
Electrons have different energies
Transmission depends on energy

$$S_S = \frac{e^2}{h}\mathcal{T}(1-\mathcal{T})\cdot eV$$

$$S_N = \frac{2e^2}{h}\mathcal{T}\cdot k_B T$$

$$S_N + S_S \approx \frac{2e^2}{h}\mathcal{T}\cdot k_B T^*$$



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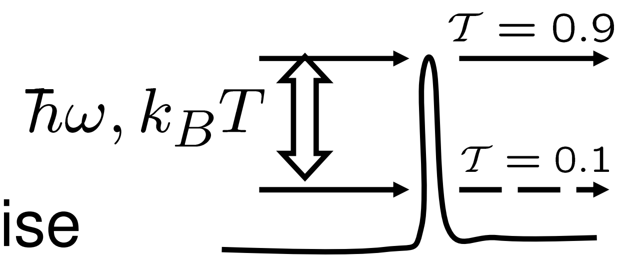
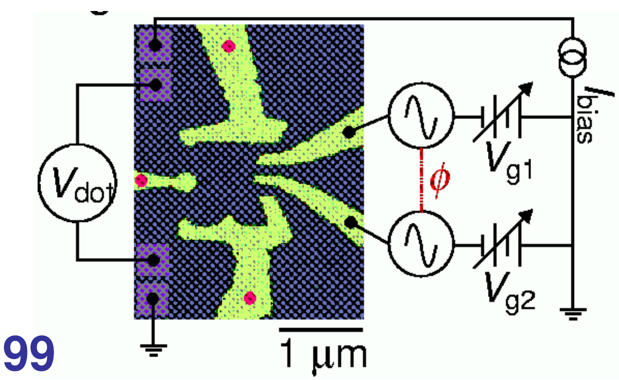


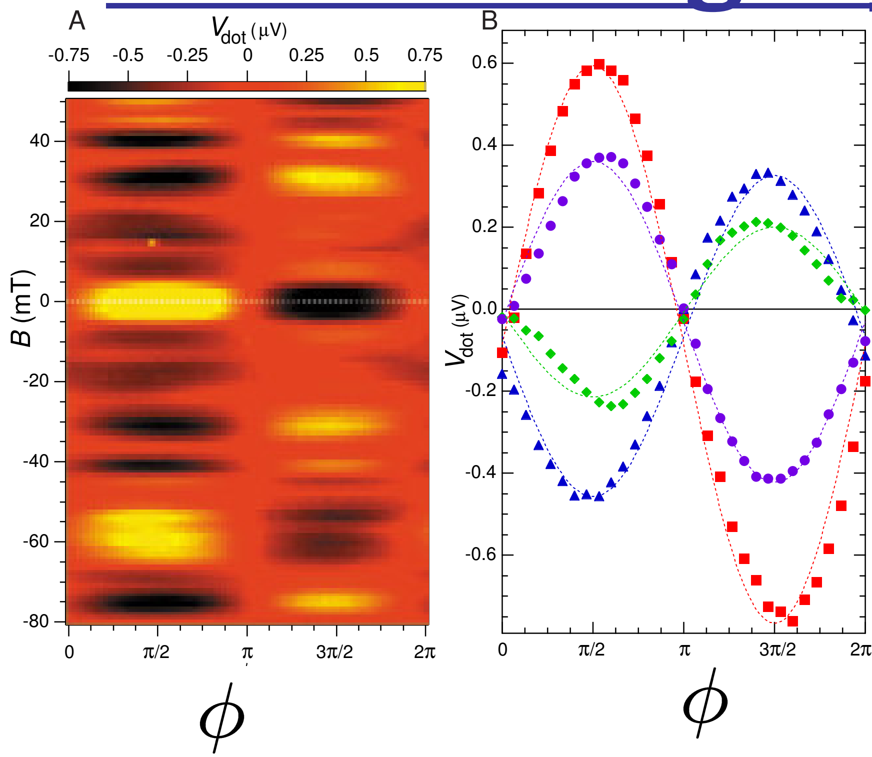
1. AC-biased quantum dot creates dc-current. Can one kill noise and reach $\mathcal{F}\to 0$?

2. Can one change the noise? Yes!

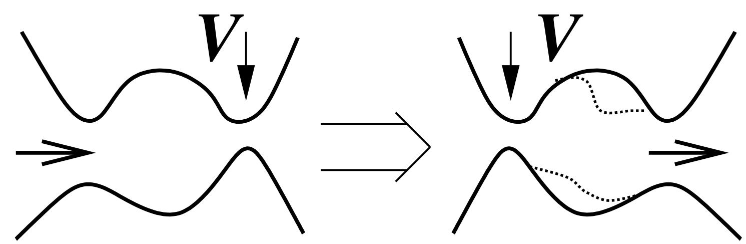


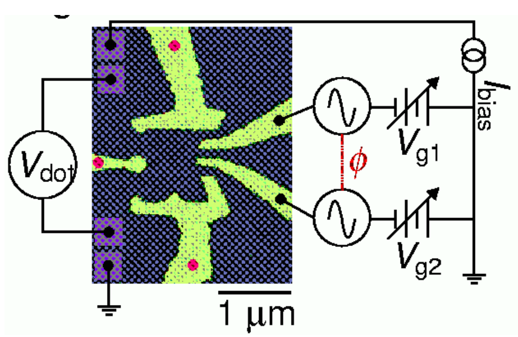
Switkes *et al* ’99

Noise through quantum pumps



Out-of-phase voltages pump electronic wave-function from one reservoir to the other

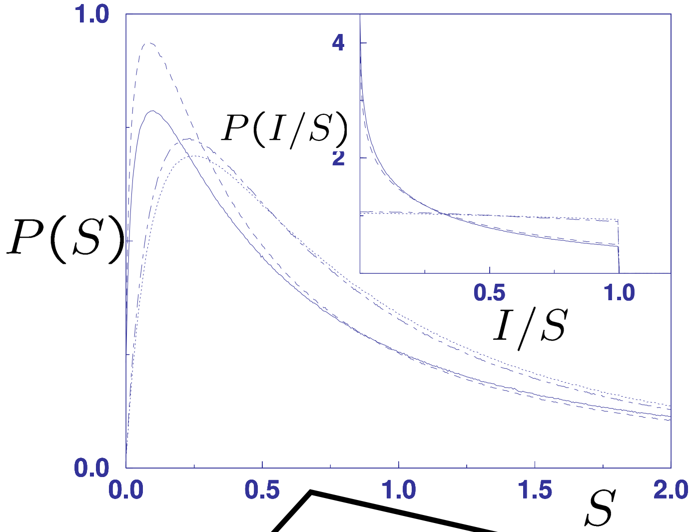


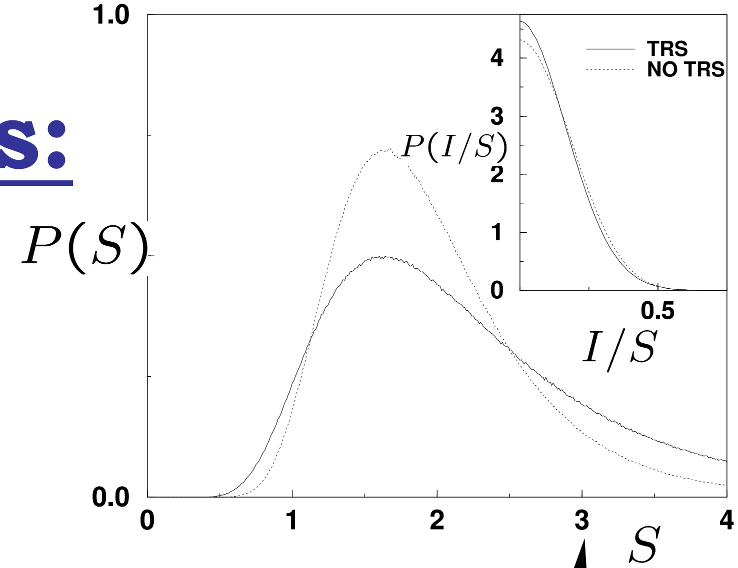


DC-current $\propto V_1 V_2 \sin\phi$ can have any sign
(Brouwer'98)

Q: Can we have zero noise?

Results:





Single-channel dot $N_L = N_R = 1$

$P(S)$ highly non-Gaussian, modified by interaction

$I/S < 1$ is limited and $\mathcal{F} = S/I > 1$

super-Poissonian!

Multi-channel dot

$N_L = N_R = 5$

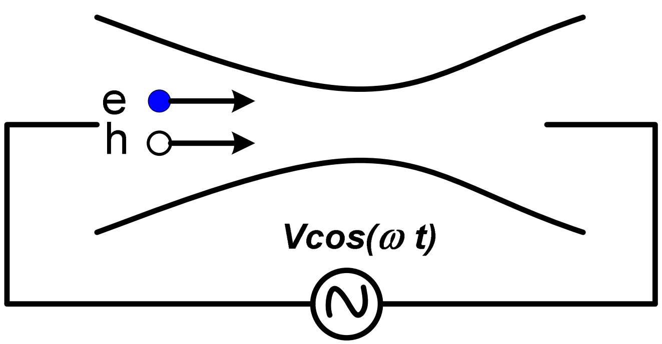
$P(S)$, $P(I/S)$ are closer to Gaussian

Polianski, Vavilov, and Brouwer '02

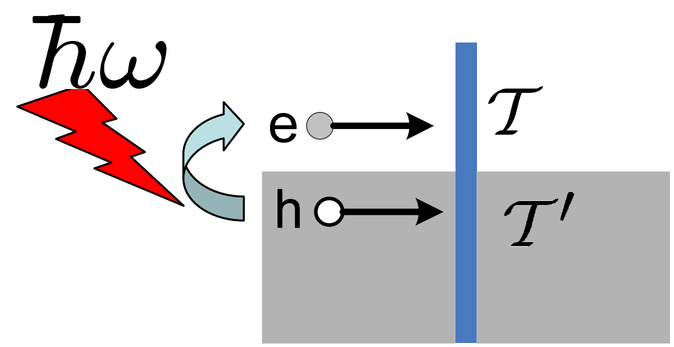
Photo-assisted shot noise

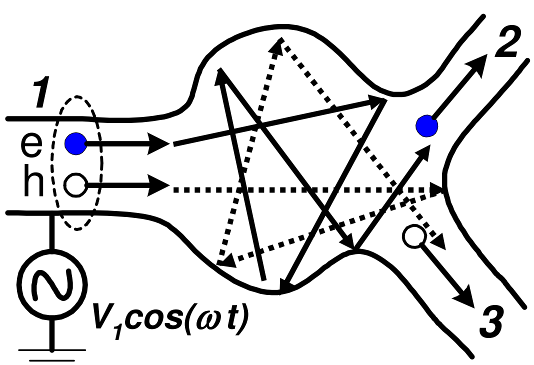
- Experiment on noise in QPC or quantum dots at low T

Reydellet et al'03

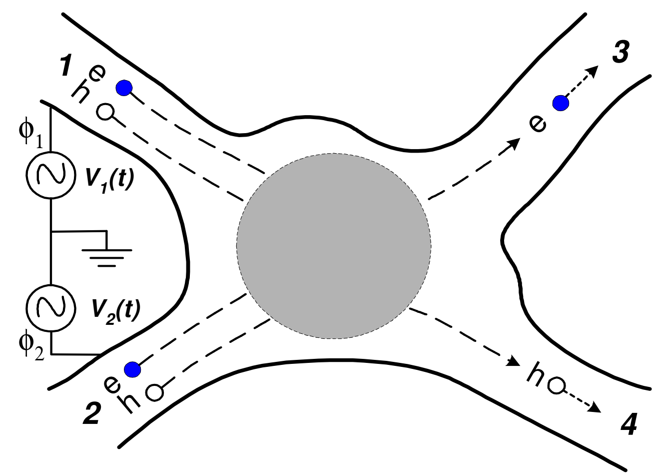


Electrons with energy $-\epsilon$ below E_F excited to $\hbar\omega - \epsilon > 0$ leaves behind a hole $-\epsilon$



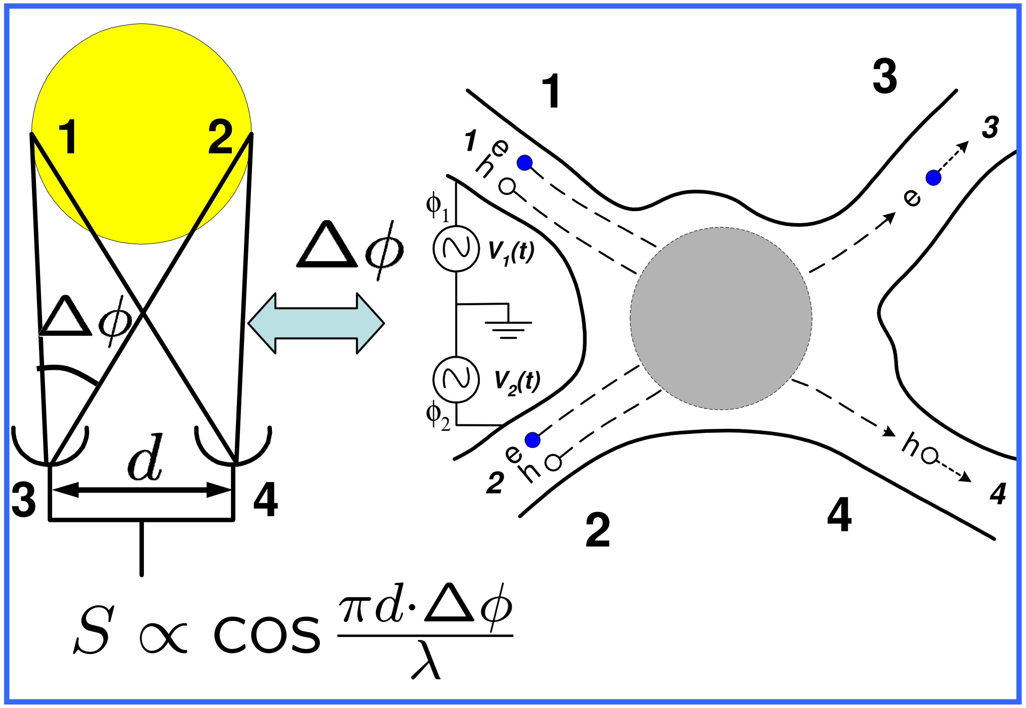


$$I_2 I_3 = -1$$



Stellar interferometry in dots

HBT-interferometer: intensity-intensity correlations from incoherent sources measure small angles $\Delta\phi$



In solid-state we can change $\Delta\phi$ ourselves!

$$V_1(t) = V\cos(\omega t)$$
$$V_2(t) = V\cos(\omega t + \Delta\phi)$$

Noise is maximized at

$$\Delta\phi = \chi = \arg\left(s_{13}^{\dagger} s_{32} s_{24}^{\dagger} s_{41}\right)$$

Rychkov, Polianski, and Büttiker '05

Variation of $\Delta\phi$ controls the correlations

What if you <u>don't</u> want noise (HDD), but include spins?

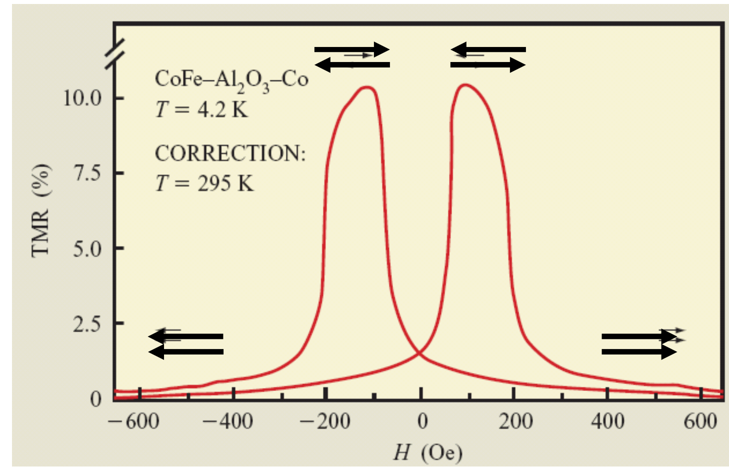
Magnetoresistance in memory

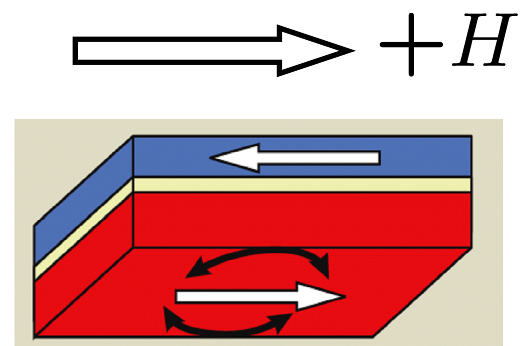
- Read/write in HDD based on magnetiresistance: different layer configurations --different resistance

Strong field $\vec{H}$ aligns others $\uparrow\uparrow$

✓ Write: field of write-head switches free layer (keep 0 or 1)

✓ Read: CPP current detects layer





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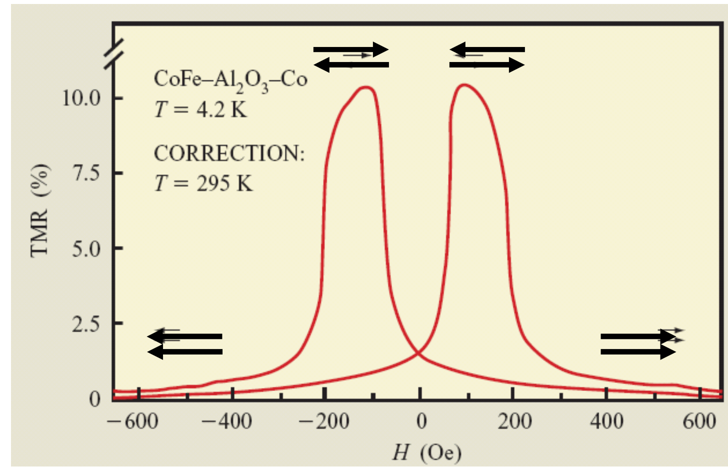
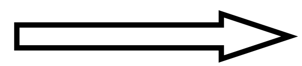
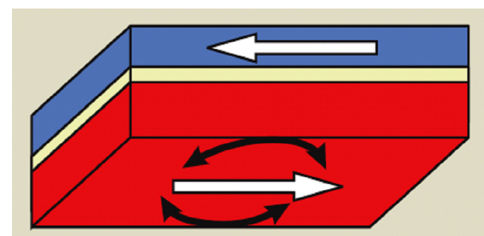
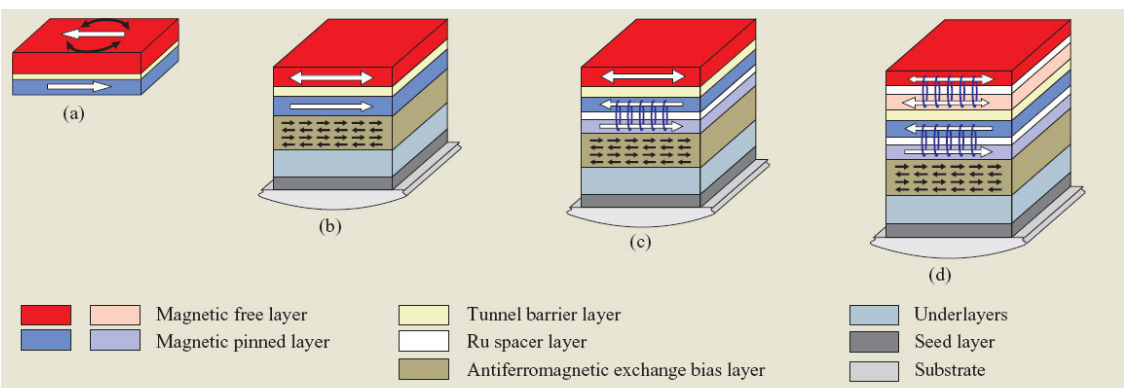
Operate when one layer is fixed





$+H$





Parkin'05

IBM J. Res. Dev (online)

Switching by current

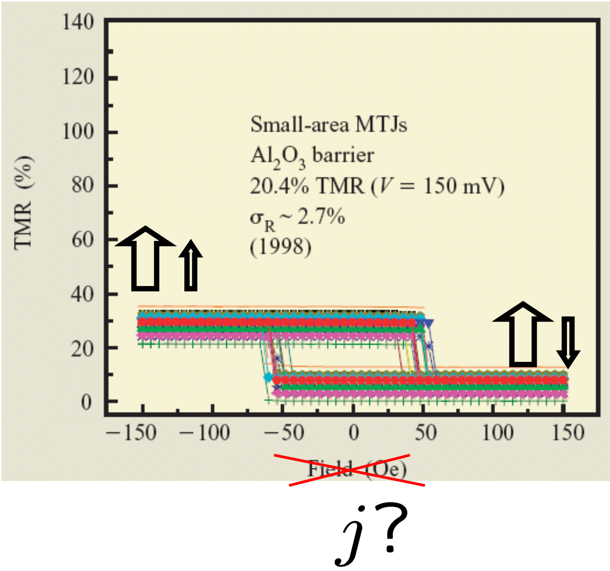
- Can current play the role of $\vec{H}$

Could we use electron spins $\vec{s}$?

Use GMR (Nobel-2007):

Filtering: Parallel $\vec{s} \uparrow\uparrow \vec{m}$ are better transmitted, then anti-parallel $\vec{s} \downarrow\uparrow \vec{m}$

Spins rotate $\vec{m}$ (spin torque)



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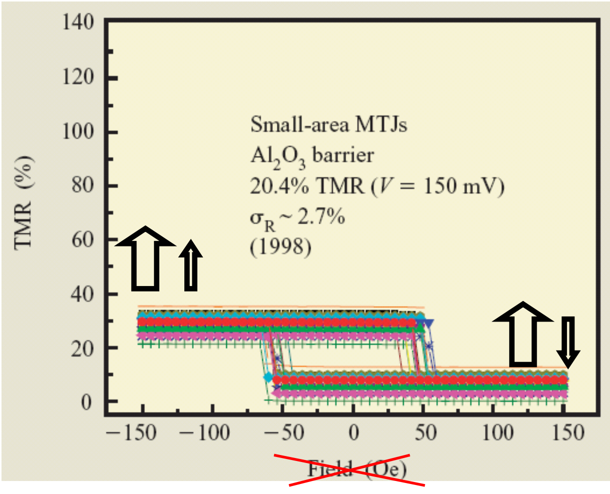
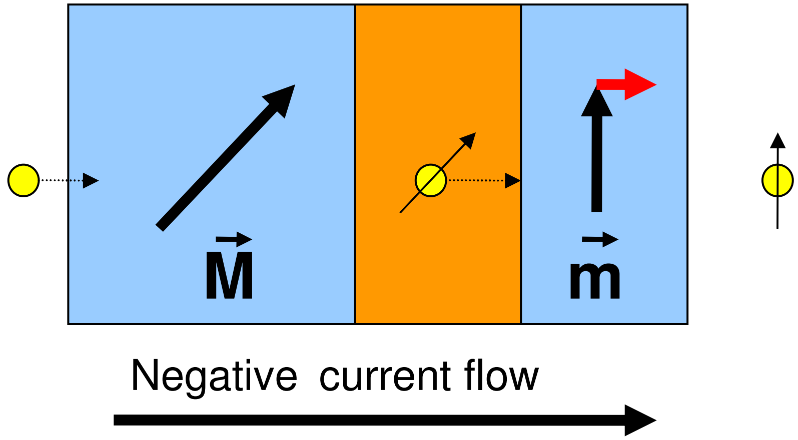
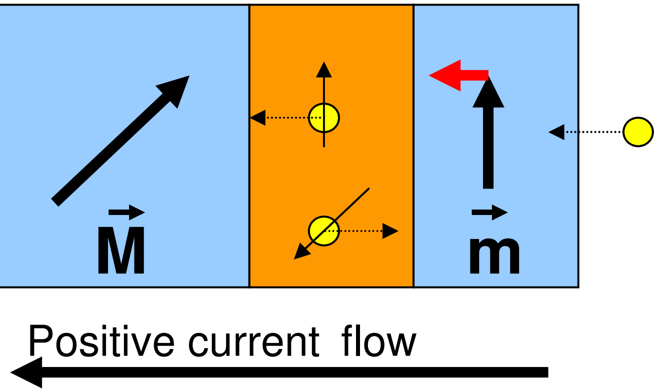
Depending on the current direction, $\vec{m}$ is rotated into P or AP configuration

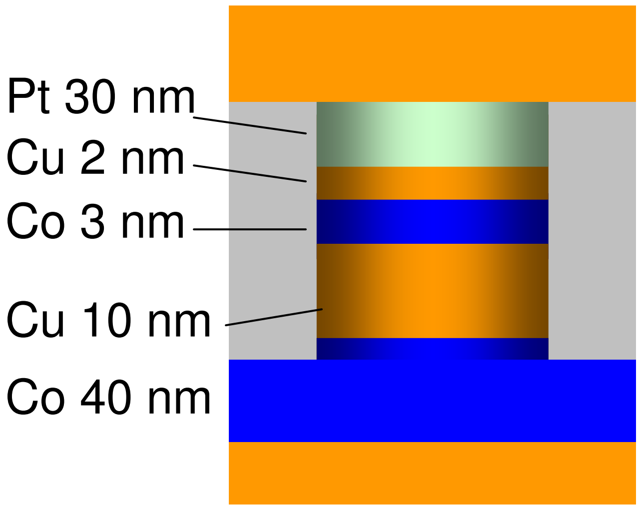


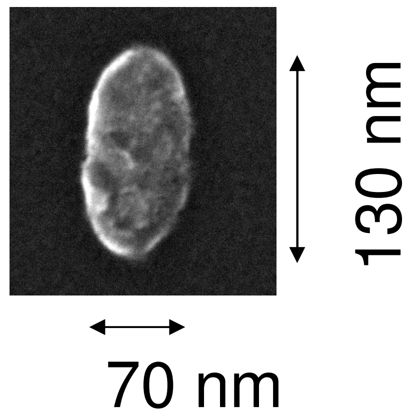
j?



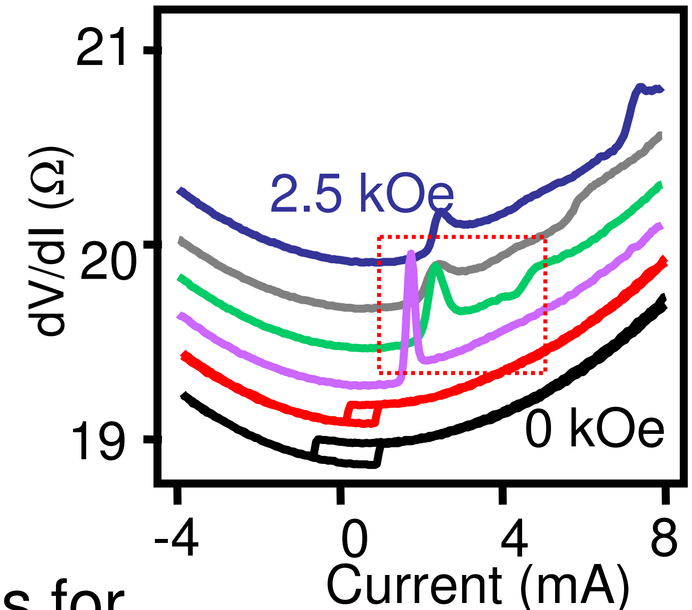


Experiment in Cornell





Kiselev *et al* '03

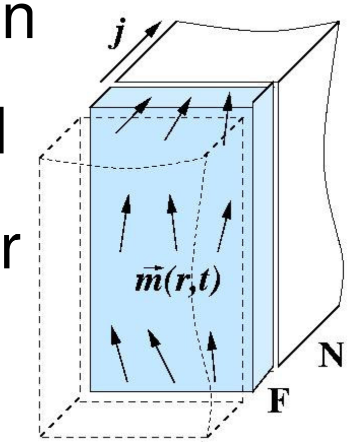


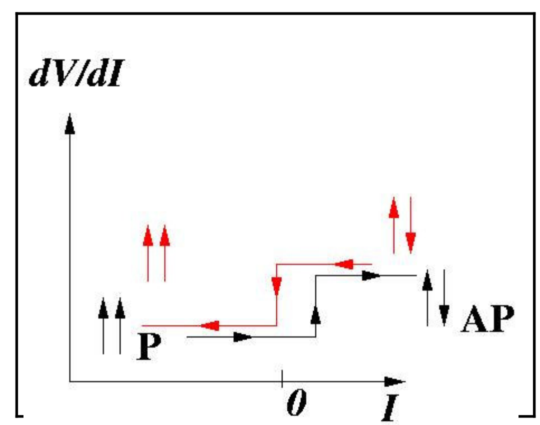
Switch from hysteresis to weird effects for one current-direction

Hysteresis: MRAM

Wave: spin-transfer

oscillators

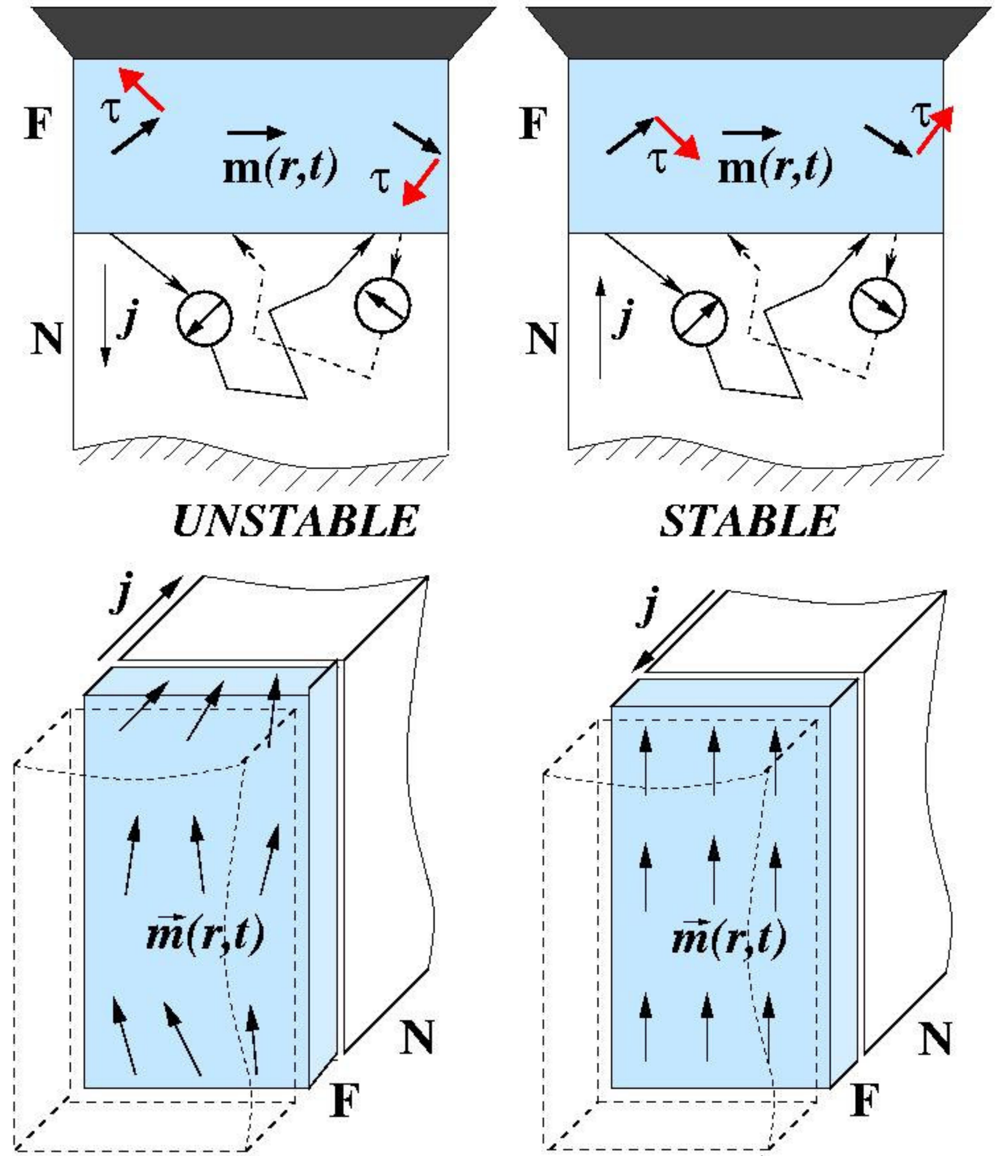




With transverse variation of m

Transverse spin diffusion of reflected/transmitted electrons in normal metal creates spin torque

Direction of current defines sign of the torque



Polianski and Brouwer '03

Conclusions

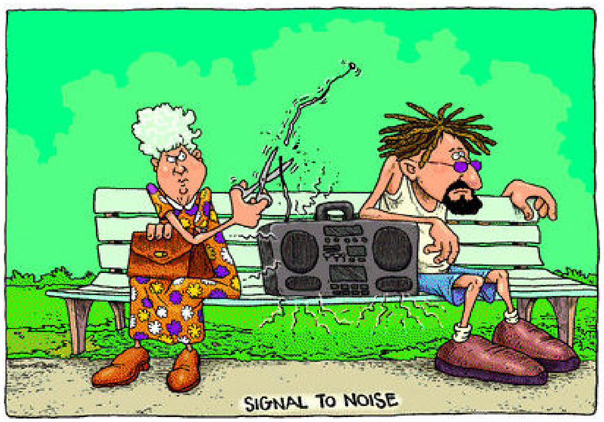
- Mesoscopics is when classical physics breaks down.

Can be 0.1 nm, can be >10 μm

- Fluctuations belong to quantum physics:

✓ Each sample is unique

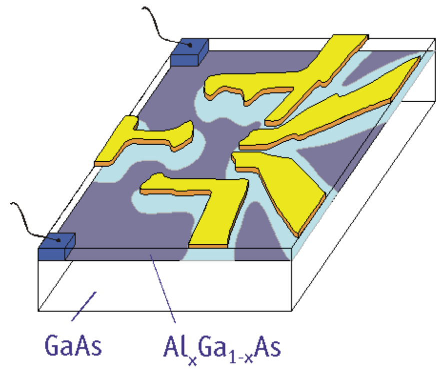
✓ Electrons are unique too

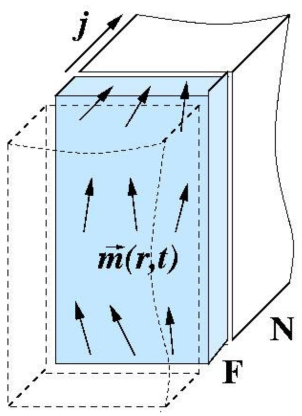


- Noise gives information unavailable from current measurements

(interactions, decoherence)

- Quantum mechanics can be useful in applications (spins)





A lot more is left to do…

Recommended Reading

- **Quantum Transport in Semiconductor Nanostructures,**

  Beenakker and van Houten, *Solid State Physics **44**, 1 (1991)* or ***cond-mat/0412664*** (general review of 2D mesoscopics)
- **Electronic transport in mesoscopic physics** Datta *(1995)* ***MIPT library*** (popular book on mesoscopics)
- **Введение в мезоскопическую физику**, Имри (2004) ***MIPT library*** (обзор мезоскопической теории)
- **Concepts in spin electronics,** Maekawa (ed.) *(2006)* ***MIPT library*** (modern concepts in spintronics)
- **Shot Noise in Mesoscopic Conductors,** Blanter and Büttiker, *Phys. Rep.* ***336,*** *1 (2000)* or ***cond-mat/9910158*** (general review of noise)
- **Random-matrix theory of quantum transport,** Beenakker, *Rev. Mod. Phys.* ***69****, 731 (1997)* or ***cond-mat/9612179*** (review of dots, wires, superconductors)
- **Quantum Shot Noise** , Beenakker and Schonenberger, *Physics Today, 37 (May 2003)*, ***cond-mat/0605025***
- **The Statistical Theory of Mesoscopic Noise**, Levitov , ***cond-mat/0210284***

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